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Algebraic generalization of the cash flow statement: Reflections by means of an algebraic algorithm

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Abstract

Starting on January 1, 2008 it became mandatory for all Brazilian public companies and private companies with net worth greater than two million reais (about one million dollars as of this writing) to publish a cash flow statement (CFS) as part of their financial statements, making this statement another important source of information for investors. This article proposes an algebraic generalization for the CFS. Working papers can help fill in a gap in teaching about cash flow statements and produce an indirect method and a direct method, side by side with their equivalence highlighted, in a single matrix by means of algebraic algorithms. This study is normative in nature and stresses the transversal relationship between accounting and mathematics, showing that accounting reports and their structures can be seen as matrices and be subjected to algebraic deductions about the events recorded by double entries. As a result, we demonstrate a mathematical algorithm with matrices and submatrices and a script in the format of working papers, compatible with the normative orientations of the Federal Accounting Council (CFS) and Brazilian legislation, permitting formulation of clear, reliable and effective cash flow statements.

Keywords: Algebraic algorithm, CFS, double-entry bookkeeping

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1. INTRODUCTION

The idea to debit x in account A and credit x in account B every time that x is the value of entry in the daybook is a notable algebraic model created by unknown accountants and mainly formalized by Luca Pacioli (PACIOLI, 1494), who wrote in his work *Tratactus de Computis et Scripturis*, or Double-Entry Bookkeeping: "... the accounting idea of debits and credits corresponds to the theory of positive and negative numbers."

This logically leads to the fundamental accounting equation:

$$\textbf{Assets = Liabilities + Stockholders' Equity} \quad [1]$$

The applicability of algebra is thus well established in the very foundation of accounting, as well as its influence in the preparation of financial statements formalized in matrices of algebraic models. The usual accounting ledgers and statements are unthinkable without their matrix structure. As early as the eighteenth century, Jean Le Rond D'Alembert (1717 – 1783) called attention to the potential application of algebra (MACHALE, 1993), recognizing that: "*Algebra is generous: she often gives more than is asked for*." The same goes for accounting, based on the algebra of double entries.

There is a didactic gap in the teaching about cash flow statements, because there is no clear, secure and effective method for teaching the preparation of this statement (MARQUES; CARNEIRO & KUHL, 2008; CAMPOS, 1999; FIPECAFI, 2010). Basically, these sources only offer examples of cash flow statements. The student will surely face difficulties in preparing a CFS by following the examples provided. One of the main problems is the apparent "mystery" involving the equivalence between the conciliation of the net income in the indirect method and the operational payments and receipts in the direct method. The algorithm proposed here aims to provide a clear, secure and effective method to teach this to accounting students.

More specifically, we propose an algebraic method materialized in working papers for CFS preparation, where the students visualize the two forms of CFS as only being two expressions of a single equation. Consequently, it is not natural to envision them as separate and independent, as the literature in general appears to suggest. We do not cover the inaccuracies related to the definition of operational activities, which are obviously related to inaccuracies in the definition of investment and financing activities. Marques, Carneiro & Kuhl (2008) contains a meticulous description of CPC Pronouncement 03, covering the problem in a thorough manner.

In short, our objective is to offer a didactically more efficient and effective method to prepare the CFS, free of particular examples, which are always insufficient for different accounting presentation systems, by means of algebraic analysis and an algorithm.

2. THEORETICAL FOUNDATION

The publication of the CFS became mandatory in Brazil for all listed companies and firms with net worth greater than two million reais according to Law 11,638/2007 and the follow-on CVM Declaration 547/2008, which approved CPC 3.

Article 1 of Law 11,638/2007 altered Law 6,404/1976 (Law of Corporations) by giving new wording to its Article 176, including numeral IV, making publication of the CFS mandatory for the described companies. Then, CVM Deliberation 547, published on August 15, 2008, approved CPC 3, which covers the rules for preparing the CFS.

CPC 3 – CFS is analogous to IAS 7 from the International Accounting Standards Board (IASB). In its 26 pages it establishes the content, objectives, scope and benefits of cash flow information, along with definition of cash and cash equivalents, presentation of operational/investment/financing activities, disclosures and other instructions on preparing cash flow statements by financial and nonfinancial firms and their direct and indirect models (http://www.cpc.org.br, fev/2010).

Since it is still a recent report in Brazil, only required as of 2008, the CFS has been prepared in companies and schools according to each one's abilities. The direct method, although apparently easier, in reality requires precision of the available accounting systems and for this reason this statement is often prepared from electronic spreadsheets and through trial and error (KASSAI, 2009).

Therefore, the approach to the CFS proposed here emphasizes the algebraic properties of [2] and [3], in their invariant character. Based on them, column matrices with invariant sums can be used to demonstrate the cash flows by means of an algebraically justified algorithm, which supplies the direct and indirect methods mentioned in CPC 3 at the same time.

Evolution itself is viewed by the science of complexity (BEINHOCKER, 2006, p. 317) as a learning algorithm: "Evolution is a knowledge-creation machine — a learning algorithm."

The accounting literature, at least that intended to be didactic, should present, with clarity and distinction, incorporating the recommendation made by Descartes for all sciences (DESCARTES, 1637), working papers accompanied by algorithms for making the entries, to produce matrices systematically and securely from which the accounting statements can be formulated. In general, textbooks use numerical examples to explain the statements, but all it takes is for a company to appear that does not have the same account groups for a difficulty to arise that immediately interrputs the capacity to prepare the report with the same efficiency and efficacy.

A characteristic of this algebraic model is the consideration that the two methods to prepare the CFS (direct and indirect) are algebraically connected, not only because two of their three matrix structures coincide (investment and financing), but mainly because they have intrinsic algebraic symmetries resulting from the algebra of double entries.

The entries in the daybook are the "fundamental particles" of accounting. The definition of the direct method is clear and distinct: only entries of the "payment" and "receipt" types should appear in the CFS. Analogously, a definition of the indirect method should clearly and distinctly specify which entries need to be evidenced. The problem is restricted to the first of the three matrix structures of the CFS, the set of operating activities, since for the investing and financing activities the definition is the same in both methods. The imprecision rests exactly in the definition of how to conciliate net profit with cash flow. Algebraically speaking, eliminating the imprecision of the "conciliation of net income" and eliminating the imprecision in the choice of the accounts whose variations must be evidenced are the same thing.

The difficulty of preparing the CFS does not end with the doubt about "where to start?" It extends to knowing how to verify easily and systematically whether or not the CFS is correct, which involves knowing whether or not the totals are right and how to be sure of this. For this reason, the algorithm proposed here has an algebraically natural characteristic, which enables verification of all the relations involved in the CFS.

3. METHODOLOGY

This article explores the difference algebra of two consecutive balance sheets, expressed in the equations:

$$\mathbf{\Delta A = \Delta L + \Delta E} \qquad [2]$$

$$\mathbf{\Delta CashEq} = -\Sigma\mathbf{\Delta}[\text{asset accounts}] + \Sigma\mathbf{\Delta}[\text{liability accounts}] + \Sigma\mathbf{\Delta}[\text{equity accounts}] \qquad [3]$$

Where:
Δ = difference
CashEq = cash equivalents
Σ = sum, combined with a specific matrix structure.

Equations [2] and [3] are nothing new to accountants (MARQUES, CARNEIRO & KUHL, 2008), although perhaps their interpretation as algebraic equations derived from the fundamental accounting equation [1]. Equations [2] and [3] appear in a promising way, but we leave their algebraic potential untouched for now and instead follow a path of "examples" to explain the CFS (MARQUES, CARNEIRO & KUHL, 2008; CAMPOS, 1999; FIPECAFI, 2010). But is also needs to be presented directly, without subterfuges, and to prevail logically for itself.

With respect to the scientific approach, we stress interdisciplinarity, in the sense that accounting reports are matrices with a certain structure and algebraic deduction, based on the definition of double entries, both drawing inspiration from the Cartesian method (DESCARTES, 1637), which is applicable to all systems that are intended to be scientific. This option for scientific approach stresses the mathematical posture of Luca Pacioli of basing accounting on the algebraic properties of positive and negative numbers, is subordinated to the philosophical method of Descartes (DESCARTES, 1637) and relies on the potential of algebra according to the vision of D'Alembert (MACHALE, 1993).

Regarding methodology, this is a normative study, since it suggests how the CFS should be prepared by means of matrices and in the form of working papers.

An analysis of the articles and books cited on the CFS them show, on the one hand, the application of the ideas developed here, and on the other provides a sample of their absence in the available literature.

For example, it is not likely that a typical student will learn to prepare a CFS by the arithmetic example given in the *Manual de Contabilidade Societária* (FIPECAFI, 2010). The same comment applies to all seven previous editions of this important manual. It is also unlikely that an example can be generalized for any other accounting plan. Other texts cited here have the same difficulty (MARQUES, CARNEIRO & KUHL, 2008; CAMPOS, 1999).

The foreign literature does not appear to be any exception (NURNBERG, 1989; *DRTINA & LARGAY, 1985*). There are works containing extensive research on cash flow, but the focus on the subject, besides not being elementary as in the present work, is completely different.

For example, the work of Patrícia Dechow (DECHOW, 1994) contains various articles addressing the interesting question of measuring the important intangible "firm performance", for which she employs cash flows as measurable indicators. In DECHOW, KHOTARI & WATTS (1998), the authors investigate the problem of predicting cash flows by a mathematical-statistical model involving time series, a theme that is far from being trivial and a is far cry from the elementary theme presented here in providing a clear understanding of how to prepare a reliable and effective cash flow statement.

It is difficult to find in the literature, either in Brazil or abroad, articles or books covering the CFS preparation in a way similar to that adopted here. This was our experience in trying to describe a type of state of the art regarding this subject. Any relevant information in this respect would be very welcome.

A tradition that has been expanding in accounting is the use of arithmetic examples to present and explain accounting concepts and theories. However, the nature of accounting is algebraic, as astutely observed by Luca Pacioli in relating credits and debits to positive and negative numbers. Therefore, the ultimate fundamental truth of accounting is expressed by the algebra contained in ledger matrices, determined by the principle of double entries, combined with traditional logic: principle of no contradiction, principle of non-contradiction, principle of excluded third, calculation of propositions, truth tables, Aristotle's syllogism, rules of deduction such as Modus Ponens and Modus Tollens, implications, equivalencies, etc., and even more, inspired by the Cartesian ideal of clarity and distinction introduced by Renée Descartes (born on March 31, 1596 in La Haye, today Descartes, Touraine, France, and died on February 11, 1650 in Stockholm Sweden) (DESCARTES, 1637):

> The first rule was never to accept anything unless I recognized it to be evidently such: that is, carefully to avoid precipitation and prejudgment, and to include nothing in my conclusions unless it presented itself so clearly and distinctly to my mind that there was no occasion to doubt it.
> The second rule was to divide each of the difficulties which I encountered into as many parts as possible, and as might be required for an easier solution.
> The third rule was to think in orderly fashion, beginning with the things which were

> simplest and easiest to understand, and gradually and by degrees reaching toward more complex knowlwdge (...).
> The last rule was always to make enumerations so complete, and reviews so general, that I would be certain that nothing was omitted."

An argument not based on this last rule is not rigorously determinable by accounting according to the model formalized by Luca Pacioli.

4. RESULTS AND DISCUSSION

Below we present an example of a traditional accounting argumentation, which relies on arithmetic and "appears to forget" algebra, double entries and elementary logic, guided by the Cartesian ideal of clarity and distinction.

TABLE 1 [NURNBERG, 1989; *DRTINA & LARGAY, 1985]*
DRTINA AND LARGAY ILLUSTRATIONS

Panel 1 — Assumptions

Schedule of Production (Physical Units)	
Beginning inventory	3,000
Add: Production for period	5,000
Total available	8,000
Less: Sales for period	4,000
Ending inventory	4,000
Cost per Manufactured Unit	
Variable – direct materials, direct labor, variable overhead – all out-of-pocket	$ 2.00
Fixed – all depreciation ($ 5,000/5,000 units produced)	$ 1.00
Total	$ 3.00
Other	
No change in work-in-process, receivables, or payables	

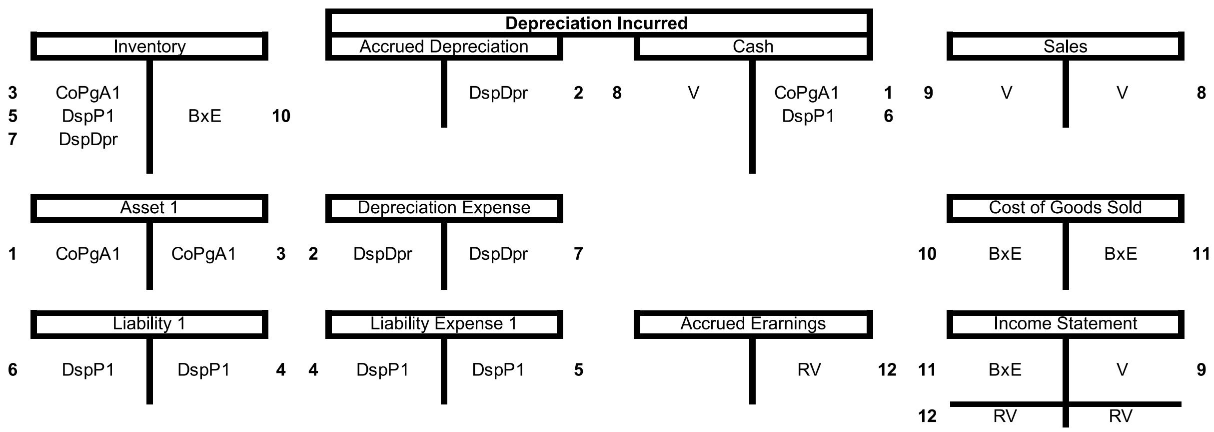
Panel 2 — Calculation of Cash Flow from Operations

Direct Method	
Collections (4,000 units sold @ $ 5)	$ 20,000
Payments (5,000 units produced @ $ 2 variable manufacturing cost)	$ 10,000
Cash flow from operations (correct)	$ 10,000
Indirect Method	
Sales (4,000 units @ $ 5)	$ 20,000
Cost of sales — LIFO (4,000 units @ $ 3 full cost)	$ 12,000
Net income	$ 8,000
Add: Depreciation expensed in cost of sales (4,000 units @ $ 1)	$ 4,000
Working capital provided by operations	$ 12,000
Less: Increase in inventory (1,000 units @ $ 3)	$ 3,000
Cash flow from operations (incorrect)	**$ 9,000**

TABLE 2 [NURNBERG, 1989; *DRTINA & LARGAY, 1985]*
EXTENSION OF DRTINA AND LARGAY ILLUSTRATIONS

Panel 1 — Indirect Method	Drtina-Largay	Alternative
Sales (4,000 units @ $ 5)	$ 20,000	$ 20,000
Cost of sales — LIFO (4,000 units @ $ 3 full cost)	$ 12,000	$ 12,000
Net income	$ 8,000	$ 8,000
Add: Depreciation incurred for period (5,000 units @ $ 1)	$ 5,000	
Add: Depreciation expensed in cost of sales (4,000 units @ $ 1)		$ 4,000
Working capital provided by operations	$ 13,000	$ 12,000
Less: Increase in inventory (1,000 units @ $ 3)	$ 3,000	
Less: Increase in inventory net of depreciation capitalized therein (1,000 units @ $ 2)		$ 2,000
	$ 10,000	$ 10,000

Panel 2 — Direct Method	Drtina-Largay	Alternative
Sales (4,000 units @ $ 5)	$ 20,000	$ 20,000
Less: Increase in receivables	- 0 -	- 0 -
Cash receipts from operations	$ 20,000	$ 20,000
Cost of sales – LIFO (4,000 units @ $ 3 full cost)	$ 12,000	$ 12,000
Less: Depreciation incurred	(5,000)	
Less: Depreciation expensed		(4,000)
Add: Increase in inventory (1,000 units @ $ 3)	$ 3,000	
Less: Increase in inventory net of depreciation capitalized therein (1,000 units @ $ 2)		$ 2,000
Less: Increase in payables	- 0 -	- 0 -
Cash payments for production	$ 10,000	$ 10,000
Cash flow from operations	$ 10,000	$ 10,000



A scenario that is more coherent with the algebraic nature of this problem of depreciation in manufacturing firms uses the algebra that is implicit in ledger matrices. "S" is the notation for "any sale fixed for analysis". This is a subtle theoretical point since "S" is a variable, because it indicates any sale with the property of being fixed for analysis. This theoretical posture is mathematically superior to "consider a sale of \$ 20,000". This latter is analogous to affirming that "the order of the terms does not alter the sum because $2 + 3 = 3 + 2$". The algebraic posture affirms that "the order of the terms does not change the sum because $U + V = V + U$, for any U and V". Therefore, the equality "$2 + 3 = 3 + 2$" does not explain the invariance of the sum by the order of the terms. Much to the contrary, it is explained by the invariance of the sum irrespective of the order of the terms. The algebraic posture creates a clearer and more distinct accounting scenario. Besides this, the algebraic notation eliminates the loss of arithmetic generality of the approach of Nurnberg, Drtina & Largay (1993), and the ledger matrices permit rigorous verification, with clarity and distinction, of whether the principle of double entries was applied correctly. Finally, calling on classic elementary logic, it is possible to trace the entries and understand the reason why the error indicated by Nurnberg, Drtina & Largay occurred. These authors attribute the error to a "mechanical application" of the indirect CFS method. But they do not explain what a "mechanical application" means. An algebraic analysis clarifies the problem, with the above matrix being accompanied by a logical script to follow and the extraction from [3] of the terms involved in the problem.

Definition 4.1. A subvariation is any entry recorded in the ledger for a period. A variation is any difference between balances of an account from two consecutive balance sheets.

Definition 4.2. A partition $\wp[C] = \{C_1, C_2, ..., C_n\}$ of a set C is a set of disjoint subsets $C_1, C_2, ..., C_n$, of C such that $C = \cup_i C_i$.

Definition 4.3. Let S be a set of subvariations in a period. Define:

$$\begin{aligned}\mathbf{\Delta CashEq}\ [S] = & -\{\Sigma\ [x \mid x \in S \text{ is a debit in assets}] - \Sigma\ [x \mid x \in S \text{ is a credit in assets}]\} \\ & + \{\Sigma\ [x \mid x \in S \text{ is a credit in liabilities}] - \Sigma\ [x \mid x \in S \text{ is a debit in liabilities}]\} + \\ & + \{\Sigma\ [x \mid x \in S \text{ is a credit in equity}] - \Sigma\ [x \mid x \in S \text{ is a debit in equity}]\}\end{aligned}$$

Theorem of additivity of ledger matrices [TALM]: Let C be a set of subvariations in a period and $\wp[C] = \{C_1, C_2, ... , C_n\}$ be a partition of C. Then,

$$\mathbf{\Delta CashEq}\ [C] = \Sigma_i\ \mathbf{\Delta CashEq}\ [C_i].$$

Equation [3] is a particular case of the **TALM** where the maximum partition of the set of subvariations in the period was considered, that is, each subvariation formed a unitary subset.

If a cash sale S occurred, then there was a stock of finished products available for sale. This eliminates the loss of generality present in the treatment of variables as constants. A purchase of material to be manufactured, represented by the variable $CoPgA1$ of **Asset 1** (entry 1), was made in cash. There is no loss of generality in assuming that only one asset is involved, that is, raw material, since the indices 2, 3, ..., could be utilized in many others, and the **TALM** would apply.

From reading the article of Nurnberg, in the case of depreciation incurred, it can be inferred that **Asset 1** depreciated by $DepExp$ (entry 2), and was transformed into a finished product (entries 2, 3, 4, 5, 6 and 7). The inventory account received the credit $CoPgA1$ in counterpart with **Asset 1**, the credit $DspP1$ in counterpart with **Liability 1** — there is no loss of generality in assuming that this liability is wages, because once again the TALM would be applied to other liabilities indexed by 2, 3, ... — and the credit $DepExp$ in counterpart with **Depreciation Expense** by the interpretation of the hypothesis that the depreciation was incurred in the period. The sale S was debited in cash (entry 8) in counterpart with the income account **Sales**. To study the contribution of this sale to the CFS, it must be transferred, by means of double entries, to the income statement (entry 9). The cost of the product sold must be transferred to the income statement (entries 10 and 11) and the contribution of this sale to net operating profit must be computed and the sale result *SR* must be transferred to Accrued Earnings (entry 12). *SR* is assumed to be positive without loss of generality because it is easy to imagine the analogous algebraic configuration if *SR* were negative. In turn, **ΔCashEq[*S*]** generates a "part" CFS[*S*] of the CFS. By the **TALM**, **ΔCashEq** is the sum of all the terms **ΔCashEq[*subvariation*]** generated by all the subvariations in the period related to *S*. By extracting from [3] only the set *C* of subvariations involving *S*, we obtain:

ΔCashEq[*C*] = – Σ Δ [asset account of *C*] + Σ Δ [liability account of *C*] + Σ Δ [liability account l.] [3][*C*]
ΔCashEq[*C*] = – Δ[Inventory[*C*]] + Δ[Accrued Depreciation[*C*]] + Δ[Accrued Earnings [*C*]]
ΔCashEq[*C*] = – [*CoPgA1* + *DspP1* + *DepExp* – *BxE*] – [– *DepExp*] + [+ *SR*]

Asset 1 and **Liability 1** have variation **Δ** = 0, related to *S*, and do not need to be explained in this demonstration.
ΔCashEq[*C*] = – [*CoPgA1* + *DspP1* + *DepExp* – *BxE*] – [– *DepExp*] + [+ *S* –*BxE*]
ΔCashEq[*C*] = *V* – [*CoPgA1* + *DspP1*]

The algebraic approach allows canceling terms, leaving only receipts and payments! In other words, it leaves a term CFS[*S*], related to sale *S* and corresponding to the operating activities related to sale *S*, by the direct method! However, if we examine the above equations a bit more carefully, all of them equivalent to the invariant relation [3][*C*], we can rewrite it in two equivalent forms:

ΔCashEq[*C*] = [*S* – *BxE*] – [– *DepExp*] – [*CoPgA1* + *DspP1* + *DepExp* – *BxE*]
ΔCashEq[*C*] = *NPOp*[*C*] + *DepExp*[*C*] – **ΔInventory**[*C*]

Algebra has not only provided the conciliation of net operating profit *NPOp*[*C*] with **CashEq**! *It is generous, giving more than is asked from it*: now we have obtained the term CFS[*C*], corresponding to the operational activities related to the sale *S*, by the indirect method! It is important to observe that CFS[*C*] produced the correct result because algebra demonstrates this fact clearly, so we can be "certain" that CFS[*C*] is right, and can check this deduction as many times as we want. Besides this, if there are any errors, they can be found and corrected by carefully following the deductive step.

In the analysis of Nurnberg, Drtina & Largay, the meaning of "algebraic algorithm for the CFS" is clear. The simultaneous obtainment of the two methods was no coincidence, as will be shown shortly. The reader can apply the suggested algorithm to the CFS**[*S*]** by examining the case (considering the analogous matrix) in which Nurnberg supposes that the depreciation is realized only at the time of the sale.

The algorithm expresses an algebraic accounting invariant (**ΔCashEq)** in a sequence of equivalent forms. We use the following algebraic notation to indicate the elements of the set **ΔΔ**ACCOUNT, denominated **subvariations**, grouped in the sets, denoted by **Δ**ACCOUNT , denominated **variations**. These subvariations are the **algebraic variables**. The subvariations are the variables that assume the values of the entries. Logically, a subvariation is a variable and an entry is a constant, just as *x* is a variable and 1 is a constant which can be substituted for *x*. Below we define the variables of a CFS example (note that this example is algebraic, and hence free of the numerical particularity that is present, for example, in the FIPECAFI Manual (2010).

ΔΔACCOUNT		**ΔΔACCOUNT**
ApprPrpExp	appropriation of prepaid expenses	**ΔPrpExp**
IncC	increase of capital	**ΔC**
WrOffPPE	write-off of property, plant and equipment	**ΔPPE**
WrOffBD	write-off of bad debts	**ΔTBRec**
CGS	cost of goods sold	**ΔE**
Purch	purchases	**ΔSupp, ΔE**
TBRec	trade bills receivable	**ΔTBRec**
TBCashed	trade bills cashed	**ΔTBCashed**
MscExp	miscellaneous expenses	**ΔMscExp**
DepExp	depreciation expenses	**ΔDepExp**
FinExp	financial expenses	**ΔFinExp**
PDDExp	expenses from provision for doubtful debts	**ΔPDDExp**
WExp	wage expenses	**ΔWExp**
DivD	dividends distributed	**ΔDivD**
IT	income tax	**ΔIT**
EBIT	earnings before income tax	
GP	gross profit	
NP	net profit	**ΔAccP**
PSPPE	profits from sale of property, plant and equipment	
NL	new loans	**ΔLoans**
PDD	provision for doubtful debts	**ΔPDD**
DvdPd	dividends paid	**ΔAccP, ΔDivD**
LoanPd	loans repaid	**ΔLoans**
SuppPd	payments to suppliers	**ΔSupp**
ITPd	income tax paid	**ΔIT**
WPd	wages paid	
PrvIT	provision for income tax	**ΔIT**
RecTB	receipts from trade bills	**ΔTBRec**
FRev	financial revenue	**ΔRF**
S	sales	**ΔTBRec**

In some periods, these variables can be zero and in others they can be accompanied by new sub-variations not related above. They can always appear on the working paper presented below and be eliminated when they equal zero in the period considered, without any impairment in the preparation of the CFS. Upon separation of the accounts according to CPC 3 in to the categories Operating Activities, Investing Activities and Financing Activities, **ΔCashEq** can be expressed as the sum of variations:

(ΔTBRec) + (ΔDDsc) + (ΔPDD) + (ΔE) + (ΔPrpExp) + ΔSupp + ΔIT + ΔS +
(ΔPPE) + (ΔDprAc) +
ΔLoans + ΔC + ΔAccP =

which is equivalent, respectively, to the sum of subvariations:

$= -[S - Binc - RecTB] - [\quad - TBCashed] - [WrOffBD - PDDExp] - [Purch + CGS] - [PrpExp - ApprPrpExp] + [-SuppPd + Purch] + [-ITPd + PrvIT] + [-WPd + WExp] +$
$-[AcqPPE - WrOffPPE] - [BDpr - DepExp] +$
$[-PmtFinExp + FinExp + NL] + [IncC + \quad] + [-DvdPd + NP] =$

NP is carried over to the start of the row and is replaced by the sum equivalent to it given by the income statement, highlighting the terms that will cancel:

$$= NP - [\mathbf{S} - \mathbf{Binc} - RecTB] - [\quad - TBCashed] - [\mathbf{WrOffBD} - \mathbf{PDDExp}] - [\mathbf{Purch} + \mathbf{CGS}] - [PrpExp - \mathbf{ApprPrpExp}] + [-SuppPd + \mathbf{Purch}] + [-ITPd + \mathbf{PrvIT}] + [-WPd + \mathbf{WExp}] + -[AcqPPE - WrOffPPE] - [BDpr - \mathbf{DepExp}] + [-\mathbf{PmtFinExp} + \mathbf{FinExp} + \mathbf{NL}] + [\mathbf{IncC} + \quad] + [-\mathbf{DvdPd} + \_\_\_\_] =$$

$$= \mathbf{S} + (\mathbf{CGS}) + FRev + (\mathbf{MscExp}) + (\mathbf{PDDExp}) + (\mathbf{FinExp}) + (\mathbf{DepExp}) + (\mathbf{WExp}) + (\mathbf{PrvIT}) + (PSPPE) - [\mathbf{S} - \mathbf{Binc} - RecTB] - [- TBCashed] - [\mathbf{WrOffBD} - \mathbf{PDDExp}] - [\mathbf{Purch} + \mathbf{CGS}] - [PrpExp - \mathbf{ApprPrpExp}] + [-SuppPd + \mathbf{Purch}] + [-ITPd + \mathbf{PrvIT}] + [-WPd + \mathbf{WExp}] + -[AcqPPE - WrOffPPE] - [BDpr - \mathbf{DepExp}] + [-PmtFinExp + \mathbf{FinExp} + NL] + [IncC + \quad] + [-DvdPd + \_\_\_\_]$$

We defined, precisely, a subvariation with nil net effect in **ΔCashEq** as being any one that appears in this sum together with its additive opposite. Therefore, logically a subvariation with non-zero net effect in **ΔCashEq** will be any one that is not accompanied by its additive opposite in this sum. The word "net" indicates the possible cancellation of the effect on **ΔCashEq**. "Cancellation" here is only an algebraic property; it is not the same as "disappearance". Equation [3] clearly shows that <u>any entry</u> is transformed into a subvariation <u>that has a net effect on the variation **ΔCashEq**</u>. The net effect may or may not be nil. A nil net effect does not cease being an effect. Algebra permits treating all entries equally as term of the same equation. The most important feature is that all the effects, that is, all the subvariations or entries for the period, are under absolute algebraic control of the person preparing the CFS.

It is important to note that once the student has a list of subvariations of the firm's accounts, the problem of preparing the CFS becomes purely algebraic. He or she has equation [3], and equivalent forms of it, during the entire preparation of the CFS, and from the outset never loses sight of this equality deduced from two consecutive balance sheets. All the cancellations are computed, except those that involve profit from sale of property, plant and equipment:

$$FRev + \underline{PSPPE} + TBCashed - NdspA - SuppPd - ITPd - WPd + -AcqPPE + \underline{WrOffPPE} - \underline{BDpr} + -PmtFinExp + NL + IncC - DvdPd$$

By definition, a subvariation such as –BDpr, because it is not present together with its additive opposite, has a non-zero net effect on **ΔCashEq**. It must be recalled that the sale of a fixed asset relates subvariations by means of the following equation: $PSPPE = SPPE + BDpr - Bimb$. Hence, the equivalent expression of **ΔCashEq** becomes:

$$= FRev + TBCashed - NPrpExp - SuppPd - ITPd - WPd + -AcqPPE + \underline{SPPE} + -PmtFinExp + NL + IncC - DvdPd$$

The cancellation in this expression of the subvariations with zero net effect on Cash Equivalent is an algebraic fact – a consequence of double entries – so that only those that represent payments and receipts in the period remain, and a few more, apparently creating difficulties for the model. However, an interesting situation arises. There are subvariations [$PSPPE$, $BDpr$ and $WrOffPPE$] whose sum [$SPPE$] is a receipt (could be a payment). By equation [3], all subvariations that are not canceled are associated with others that are neither receipts nor payments, whose sum is the same as of payments and receipts. A subvariation that was not canceled cannot be alone in the second member of [3].

<u>**Theorem 4.4**</u> [**Theorem of the direct CFS**]: All subvariations of the fundamental variation equation that are neither payments nor receipts, not canceled by the presence of the counterpart second

entry, can be associated with a non-empty set of other subvariations whose sum is zero or a sum of payments and receipts.

The accountant has the documents necessary to identify the payments and receipts that, when added, equal the sums of subvariations. In the example, the accountant identifies the subvariations $PSPPE$, $WrOffPPE$ and $-BDpr$ with the receipt $SPPE$. Algebra provides for the occurrence of situations where various payments and receipts are sums of subvariations that remain in the second member, as well as a situation in which these subvariations never group together to form a particular payment or receipt. It is interesting to know whether these examples exist in accounting. If not, then there is no logical problem, except for the fact that *algebra often gives more than is asked for*.

It is no exaggeration to state that the algebraic treatment of the subvariations rigorously and naturally describes the demonstration of the CFS by the direct method. By this route, students no longer have any pretext to exclaim "…I don't know where to start!" Besides this, they will always know all is going well and will turn out well. Algebra offers them a clear and distinct vision, allowing them total control of all entries recorded in a period. A sequence of equalities leads them to the last element of the CFS, which is the pair of matrices containing the direct and indirect method.

There are two conceivable representations of the CFS. One is algebraic, only containing subvariations of the accounts, which can remain the same for a long time, at least until the accounting plan is altered. The other is the arithmetic CFS, which consists of the substitution in the algebraic CFS of constants that represent the entries that generate the second balance sheet. Therefore, once again D'Alembert's statement is confirmed. The algebraic CFS can be prepared one time only and furnish various arithmetic CFSs by means of mere substitution of variables by constants. There can be a greater or lesser number of subvariations, but always in the same accounts.

To derive a theorem for the indirect CFS method analogous to Theorem 4.4, it is necessary to have an $m \times n$ matrix and a structure of submatrices. By the direct method, only a 1×1 matrix, that is, a variable, is necessary to know **ΔCashEq**, and a sequence of expressions of this variable to generate the CFS. The need for matrices with rows and columns to deal with the algebra of the indirect method is a clear indicator of the greater difficulty involved in this statement by means of a sequence of "equivalent matrices" where the last is precisely the CFS by the indirect method. The matrix called "**CFS**" has a structure that accommodates, in its submatrices, entries to record the amounts that refer to Operating Activities (matrix **OPACT I**, matrix **OPACT D**), Investing Activities (matrices **INVACT** and **INVACT´**) and Financing Activities (matrices **FINACT** and **FINACT´**), besides the matrices **income statement**, **ΔBP**, **ΔΔBP** and **AJLL**.

The matrix below can be used as a **working paper** for readers to insert the values according to the definitions of the CFS according to CPC 3. Note that one submatrix, called **ΔBP**, is immersed in the **CFS**, containing as entries the variations of the accounts related in the equation equivalent to [3]:

$$\begin{gathered}(\mathbf{\Delta TBRec}) + (\mathbf{\Delta DDsc}) + (\mathbf{\Delta PDD}) + (\mathbf{\Delta E}) + (\mathbf{\Delta PrpExp}) + \mathbf{\Delta Supp} + \mathbf{\Delta IT} + \\ + \mathbf{\Delta S} + (\mathbf{\Delta PPE}) + (\mathbf{\Delta DprAc}) + \mathbf{\Delta Loans} + \mathbf{\Delta C} + \mathbf{\Delta AccP} = \mathbf{\Delta CashEq.}\end{gathered}$$

Therefore, the column containing the variations **Δ**ACCOUNT with adequate sign has the sum **ΔCashEq**. The matrix **ΔCAHSEQ** is juxtaposed below the matrix **CFS** to record the correct calculation of the variation of the Cash Equivalent account, denoted by **CashEq**. The filling in of **ΔBP** is the first of a sequence that represents the invariant **ΔCashEq** expressed in forms equivalent to [2]. This is the starting point of the practical CFS algorithm.

Definition 4.5 Given two consecutive balance sheets, the difference matrix **ΔBP** is an $n \times 2$ matrix containing the variations of the accounts in the period considered. The number n of rows is the number that is sufficient to contain all the accounts separated in the **OPACT I**, **OPACT D**, **income statement**, **INVACT** and **FINACT** matrices as required by CPC 3. The column matrix **ΔΔBP** contains the subvariations that affect the accounts in the period considered. **ΔCashEq** is automatically the sum of the right-hand column. The starting point of this sequence is the equality

$$\Delta\mathbf{CashEq} = (\Delta\mathbf{TBRec}) + (\Delta\mathbf{DDsc}) + (\Delta\mathbf{PDD}) + (\Delta\mathbf{E}) + (\Delta\mathbf{PrpExp}) + \Delta\mathbf{Supp} + \Delta\mathbf{IT} + \\ + \Delta\mathbf{S} + (\Delta\mathbf{PPE}) + (\Delta\mathbf{DprAc}) + \Delta\mathbf{Loans} + \Delta\mathbf{C} + \Delta\mathbf{AccP} \quad [3]$$

It is exactly the first equivalence of the algorithm deduced from the second fundamental equation of accounting $\Delta A = \Delta \mathbf{L} + \Delta \mathbf{E}$. In the left-hand column, that is, the left side of the algebraic variations, the arithmetic variations of the accounts are recorded. In the right-hand column, that is, in the column matrix **ΔΔBP**, the subvariations deduced from CPC 3 are algebraically recorded. The matrix **ΔCAHSEQ** records the confirmation of the invariant **ΔCashEq**. The above matrices are equivalent in the sense that their rows are equivalent, and thus the sum of the rows is the same, exactly equal to **ΔCashEq**, the value of the CFS that needs to be "demonstrated". The algorithm will maintain the "certainty" that the CFS will be correct because the sequence always produces the sum **ΔCashEq**, and hence if the last matrix satisfies the CFS definition of CPC 3, then the CFS will be correct. Besides this, the algorithm permits as many verifications as necessary for the reader to be "convinced" that the resulting CFS is correct. The most important characteristic of this algorithm is that it allows the reader to efficiently search for the origin of any errors, such as failing to obtain the correct sum **ΔCashEq** in any of the steps, not only in the last step, which produces the CFS.

The previous step shows clearly what the algorithm's "intention" is and why it is called "algebraic". The *NP* is taken from the row **ΔAccP** and carried over to the row of the matrix **AJLL**, without leaving the matrix **OPACT I**. Therefore, if the rows of the right-hand column of this matrix are added, taking care to include the subvariation *–Dvd*, which was shifted to the column to its right, the sum **ΔCashEq** will continue being obtained. The variation **ΔAccP** no longer appears in **OPACT I**, but still continues to contribute to the sum of the rows of the column, which is precisely the variation **ΔCashEq** of the Cash Equivalent account. The matrix **OPACT D** was altered, ceasing to be empty. It now contains the subvariations *NP* and – *Dvd*. The sum of its left-hand column is **ΔAccP** = *NP* – *Dvd*. Therefore, **OPACT D** starts to be filled in with the subvariations of **ΔΔBP,** and when all the subvariations have been carried over to it, the sum of its rows will evidently be **ΔCashEq**. The algorithm will maintain in **OPACT D** only the payment and receipt subvariations. Thus, *NP* cannot remain in this matrix, since it is not this type of entry. The algorithm, then, replaces *NP* with its subvariations from the **income statement,** whose sum is the same. In other words, the subvariations that produce the **income statement** are introduced to supply a value equivalent to the *NP*.

The row sums of the **income statement** are thus excluded, only leaving the subvariation terms. The matrix **OPACT D** was transformed into an equivalent matrix in the sense that the sum of its rows continues to result in **ΔAccP** = *NP* – *Dvd*. It now contains the subvariations equivalent to *NP*. Since subvariations that are neither payments nor receipts cannot remain in this matrix, the algorithm, for OPACT 1, obeying the CFS definitions of CPC 3.

For example, the subvariations *CGS* and *–S* are transported to **OPACT D**, leaving zero in their positions in **ΔΔBP**. The variations **–ΔTBRec** and **–ΔE** are not altered, so that the sum of the rows in **OPACT I** continues invariant. Regarding the matrix **OPACT D**, two new terms enter its rows, but cancel their additive opposites that are in the rows of the **income statement** submatrix. This cancellation was totally beneficial because these subvariations cannot remain in **OPACT D** due to the fact they are neither payments nor receipts. *–Purch* and *Purch* can be excluded from the rows of **ΔΔBP** because they cancel each other in the sum of these rows. Other analogous cancellations are possible, but the algorithm must be explained slowly to facilitate the reader's understanding. On transporting *–WPd* + *WExp* to **OPACT D**, *WExp* cancels with (*WExp*) and eliminates a subvariation that did not directly affect the Cash Equivalent account. Thus, the matrix **OPACT D** receives two more subvariations from **ΔBP**, maintains its characteristic of only containing payments and receipts for the period, and follows the sequence of transformations that will lead to the demonstration of the CFS by the direct method.

The matrix **OPACT I** serves for control purposes, where the variation **ΔS** is maintained, which equals *–WPd* + *WExp*, whose terms were carried to **OPACT D**. The reader can observe that this is the same as considering and keeping the variation **ΔS** in **OPACT I** and introducing the payment –*WPd* in **OPACT D**. In reality, the term *WExp* accompanies this payment to preserve the variation **ΔS**, but algebra performs the task of canceling it with its additive opposite (*WExp*), which entered **OPACT D** together with the subvariations of the **income statement,** which substituted *NP*. This small "miracle" harks back to the statement of D'Alembert about algebra. Therefore, the invariant **ΔCashEq** does not change, as a sum of the rows of the matrix **OPACT I**, due to this operation of eliminating two subvariations of **ΔΔBP,** taking a step in the direction of formation of the CFS by the direct method. The transport of –*ITPd* + *IT* to the matrix **OPACT D** can be described analogously to –*WPd* + *WExp*. The matrix **OPACT I** serves as a control, where the variation **ΔIT** is maintained and equals –*ITPd* + *IT*, whose terms were carried over to **OPACT D**. This is equivalent to considering and maintaining the variation **ΔIT** in **OPACT I** and introducing payment –*ITPd* in **OPACT D**. In reality, the term *IT* accompanies this payment so as to preserve the variation **ΔIT**, but algebra performs the task of canceling it with its additive opposite (*IT*), which entered **OPACT D** together with the subvariations of the **income statement** that replaced *NP*.

The transport of –*PFinExp* + *NL* + *FinExp* to the matrix **OPACT D** permits eliminating **ΔLoans** in **OPACT I**. However, since –*PFinExp* must belong to **OPACT I** according to the convention of CPC 3, this subvariation rises to form a group with *NP* in **OPACT I** and rises in **OPACT D** so that the sum of the operating activities coincides in the two methods. The transport of –*AcqPPE* + *WrOffPPE* to the matrix **OPACT D** permits eliminating **ΔPPE** in **OPACT I**. However, since *WrOffPPE* must compose *SPPE*, which will remain in the same row as – *AcqPPE* in **OPACT D**, *WrOffPPE* is canceled, giving a place for *SPPE*. To recover the equivalence of the matrices affected by the entry or *SPPE* in **OPACT D**, (*Lvimb*) is introduced in **OPACT I,** grouped with *NP* and canceling with *Lvimb*, –*BDpr* and *DepExp* in **OPACT D**, with this last operation implying elimination of **ΔDepExp**. The transport of *IncC* to the matrix **OPACT D** permits eliminating **ΔC** in **OPACT I**.

5. CONCLUSIONS

This normative study contributes to fill in a gap, by proposing a clear, secure and effective method to teach preparation of the cash flow statement. It offers an algorithm to prepare the CFS based on the algebraic relations between accounting variables arranged in matrices. As a result, it follows the CFS algorithm, applying six steps to manipulate accounting data.

Data: The data are contained in ***m*** asset accounts classified as involving operating activities, ***n*** liability accounts classified as involving operating activities, ***r*** asset accounts classified as involving investing activities and ***s*** equity accounts classified as involving financing activities.

Step 1: OPACT I is filled in by the variations Δ_i.

Step 2: ΔΔBP is filled in by the subvariations $\Delta\Delta_{i,k}$ that compose the variations Δ_i.

Step 3: The subvariation *NP* is carried from **ΔAccP** to **AJLLOp** and its equivalent expression given by the income statement is introduced in **OPACT D**.

Step 4: The non-operating subvariations $\Delta\Delta_{i,k}$, which are payments or receipts, are transported to **INVACT´** and **FINACT´**.

Step 5: The subvariations that are not payments or receipts, do not cancel and are operational are transported to **INVACT´** and **FINACT´**, are grouped in equivalent form to payments and receipts and are replaced by these new payment or receipt variables.

Step 6: The subvariations that are not payments or receipts, do not cancel and are operational are transported to **OPACT I**, are grouped with NP, and their respective Δ_i terms are excluded from **OPACT I**.

<u>**Theorem of the CFS:**</u> The CFS algorithm is logically consistent, efficient and effective to generate the direct method and indirect method simultaneously and equivalently.

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7. APPENDIX

<u>Demonstration</u> of the TALM: We have: $x \in C \Leftrightarrow x \in \cup_i C_i \Leftrightarrow x \in C_1$ or $x \in C_2$ or ... or $x \in C_n$. Therefore, separating all the subvariations of C that are from assets, liabilities or equity and are in C_i, by the associative property of addition, we have:

$$
\begin{gathered}
\Sigma_i\, \mathbf{\Delta CashEq}\ [C_i] = \\
= \{-[\Sigma a_1 - \Sigma b_1] + [\Sigma c_1 - \Sigma d_1] + [\Sigma e_1 - \Sigma f_1]\} + \ldots + \{-[\Sigma a_n - \Sigma b_n] + [\Sigma c_n - \Sigma d_n] + [\Sigma e_n - \Sigma f_n]\} = \\
= -\{[\Sigma a_1 + \ldots + \Sigma a_n] - [\Sigma b_1 + \ldots + \Sigma b_n]\} + \{[\Sigma c_1 + \ldots + \Sigma c_n] - [\Sigma d_1 + \ldots + \Sigma d_n]\} + \{[\Sigma e_1 + \ldots + \Sigma e_n] - [\Sigma f_1 + \ldots + \Sigma f_n]\} = \\
= -\{\Sigma[a_1 + \ldots + a_n] - \Sigma[b_1 + \ldots + b_n]\} + \{\Sigma[c_1 + \ldots + c_n] - \Sigma[d_1 + \ldots + d_n]\} + \{\Sigma[e_1 + \ldots + e_n] - \Sigma[f_1 + \ldots + \Sigma f_n]\} = \\
= -\{\Sigma\,[x \mid x \in C \text{ is debited under assets}] - \Sigma\,[x \mid x \in C \text{ is credited under assets}]\} + \{\Sigma\,[x \mid x \in \text{ is credited under liabilities}] - \\
- \Sigma\,[x \mid x \in S \text{ debited under assets}]\} + \{\Sigma\,[x \mid x \in S \text{ is credited under equity}] - \Sigma\,[x \mid x \in S \text{ is debited under equity}]\} = \mathbf{\Delta CashEq}\ [C]
\end{gathered}
$$

where a_1 = debited under assets of C_1, b_1 = credited under assets of C_1, c_1 = credited under liabilities of C_1, d_1 = debited under liabilities of C_1, e_1 = credited under equity of C_1, f_1 = debited under equity of C_1.

<u>Demonstration of the Theorem of the Direct CFS:</u> In equation [3], carry to the first member all the subvariations from the second member that are payments or receipts. If nothing remains in the second member, then there is nothing toe demonstrate and the theorem holds true. Assume that the new first member is not zero. Then there are at least two subvariations in the second member. Indeed, if there were only one, it would equal the cash balance from the first member, and thus would have to represent the payment or receipt that is lacking in the cash flow, which is not possible because there are no more payments or receipts in the second member. Thus, the subvariations not canceled in the second member add to the same value as in the first member, which is a non-zero cash balance. This means there are payments and receipts whose sum is the second member. In the worst of cases, all the payments and receipts of the first member satisfy the second affirmation of this theorem.

<u>Demonstration of the CFS Theorem:</u>
In Step 1, the sum of the **OPACT I** lines is **ΔCashEq**, that is,

$$
\boldsymbol{[-\Delta_1 - \ldots - \Delta_m + \Delta_{m+1} + \ldots + \Delta_{m+n}] + [-\Delta_{m+n+1} - \ldots - \Delta_{m+n+r}] + [\Delta_{m+n+r+1} + \ldots + \Delta_{m+n+r+s}]}
$$

according to equation [2]. In Step 2, **ΔΔBP** is filled in by the subvariations $\Delta\Delta_{i,k}$, which compose the variations Δ_i, and therefore the sum of its lines also equal **ΔCashEq**. In Step 3, the subvariation NP is transported from **ΔAccP** to **AJLLOp** and its equivalent expression given by the **income statement** is introduced in **OPACT D**. Therefore, the sumo f the **OPACT I** lies continues to be invariant and **OPACT D** starts to be filled in with the subvariations of **ΔΔBP**, a process that culminates with the transport of all the subvariations from **ΔΔBP** to **OPACT D,** with payments and receipts only remaining after cancellations permitted by the PD. In Step 4 the subvariations **ΔΔ**$_{i,k}$ of **ΔΔBP,** which were non-operating payments and receipts, are carried to **INVACT´** and **FINACT´**. This action fills in **OPACT D**, the matrix that will result from the direct CFS method. In Step 5, those subvariations that are neither payments nor receipts, not cancel and are not operational, are transported to **INVACT´** and **FINACT´**, grouping in equivalent fashion to payments and receipts and are substituted by these new payment and receipt variables, continuing to approximate **OPACT D** of the CFS by the direct method. In Step 6, those subvariations **ΔΔ**$_{i,k}$ of **ΔΔBP** that are neither payments nor receipts, or that cancel or are transported to **OPACT I**, if they are operating, and are grouped with the NP, and their respective Δ_i are canceled out in **OPACT I**. This action does not alter the sum of **ΔCashEq** from its lines because the subvariations of Δ_i continue to contribute with the same value to **ΔCashEq**, partly in **OPACT D** and partly in **OPACT I**, with the NP being adjusted as explained above.

			CFS MATRIX			
	OPACT I Matrix			OPACT D Matrix		
				~~S~~		
				~~(CGS)~~		
				(~~WExp~~)		
				(~~DepExp~~)		
AJLLOp				(~~FinExp~~)		
				(~~PDDExp~~)		DRE
				(~~MacExp~~)		
				~~FRev~~	FRev	300
	-3.000	(PSPPE)		~~PSPPE~~		
	-1.000	(PmtFinExp)		(PmtFinExp)		-1.000
	1.500	DepExp		~~(IT)~~		
		NP				
			ΔΔBS			
	-10.000	−ΔTBRec	−[S−WrOffBD−RecC]	−~~S~~+~~WrOffBD~~+RecC	RecC	29.500
	5.000	−ΔDDsc	−[−TBCashed]	~~TBCashed~~	TBCashed	5.000
	500	−ΔPDD	−[WrOffBD−PDDExp]	−~~WrOffBD~~+~~PDDExp~~		
	-3.000	−ΔE	−[Purch − CGS]	~~−Purch~~+~~CGS~~		
	-2.000	−ΔPrpExp	−[NDspA−AppDspA]	−~~NDspA~~+~~AppDspA~~	−NDspA	-2.600
	13.000	ΔSupp	[− SuppPd+ Purch]	− ~~SuppPd~~ ~~+Purch~~	− SuppPd	-10.000
	-700	ΔIT	[− ITPd + IT]	− ~~ITPd~~ ~~+IT~~	− ITPd	-2.000
	-7.000	ΔS	[− WPd + WExp]	− ~~WPd~~ + ~~Wexp~~	− WPd	-21.000
ΔBS	INVACT Matrix			INVACT Matrix		
	~~-5.000~~	~~−ΔPPE~~	−[AcqPPE−WrOffPPE]	−~~AcqPPE~~+~~WrOffPPE~~	−AcqPPE	-20.000
					SPPE	15.000
	~~-1.500~~	~~−ΔDprAc~~	−[BDpr−DepExp]	−~~BDpr~~+~~DepExp~~		
	~~10.000~~	~~ΔLoans~~	[−FinExpPd+NL+FinExp]	~~NL~~+~~FinExp~~	NL	10.000
	FINACT Matrix			FINACT Matrix		
	~~10.000~~	~~ΔC~~		~~IncC~~	IncC	10.000
	~~2.400~~	~~ΔAccP~~		~~− DivD~~	− DivD	-1.500
	11.700	ΔCashEq			ΔCashEq	11.700
ΔEQCX	5.600	Initial			Initial	5.600
	17.300	Final			Final	17.300